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Analysis vs Deduction

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In the paper, we consider four types of problems that naturally arise in connection with the definition of a logical inference: 1) verifying the proof of: $\Gamma \langle A_1, \ldots, A_n \rangle A$; 2) search for interesting consequences: $\Gamma \langle \ldots \rangle ?$; 3) search for the proof: $\Gamma \langle . ? . \rangle A$; 4) search for hypotheses: $?\langle \ldots \rangle A$. Modern logic focuses on the problem of finding the proof of statements. Gödel’s restrictive theorems have a direct relation to it. At the same time in real practice, the task of search for hypotheses is much more common. The main part of this work is devoted to the investigation of this problem. A target proposition $A$ is given, and it is required to find the set of premises $\Gamma$ from which it is logically deducible. The choice of suitable premises $\Gamma$ occurs on the basis of the logical analysis of proposition $A$. We distinguish six different grounds for the selection of these premises: 1) acceptance of explicit definitions for predicate and functional symbols; 2) acceptance of axiomatic definitions for predicate and functional symbols; 3) acceptance of previously proved theorems; 4) acceptance of empirically true statements; 5) acceptance of statements describing the result of some action; 6) acceptance of plausible hypotheses that may be relevant to the problem being solved. In this paper we construct a calculus that formalizes the problem of an analytic search for the justification of a thesis. Two metatheorems are proved, from which it follows that the constructed calculus really allows us to solve this type of problems.

Ключевые слова: problem solving, logical reduction, logical inference, search for proof, search for hypotheses, analytical tables, theory of definitions

This paper further develops ideas of the analytical approach to problem solving put forward by the author in [14, 15, 16]. It also draws upon ideas by C. Cellucci [?], whose views on the role of logic in the process of cognition are very close to the author’s views.

1. Logical deduction

In the opening of “Prior Analytics” Aristotle says that his investigation is dedicated to proof, and this is the subject of the science of proof.

Reasoning today is known to be the central concept of logic.

Logic can be regarded as a science of thorough reasoning methods. “Thorough” reasoning methods are meant as those that when given true antecedents, true consequences follow [12, p. 6].

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The concept of reasoning is refined within the framework of the notion of logical deduction.

We call a deduction of formula $A$ from the set of antecedents $\Gamma$ a non-empty finite sequence of formulas $A_1, \ldots, A_n (= A)$, with each of them being either one of the assumptions or a derivation from the previous formulas in the sequence according to one of the deduction rules, and the final formula in the sequence is $A$.

Under such a definition, logical deduction takes the form of three-component structure $\Gamma \langle A_1, \ldots, A_n \rangle A$:

1. Set of suppositions (hypotheses) $\Gamma$.
2. Sequence of formulas $\langle A_1, \ldots, A_n \rangle$.
3. The final formula of this sequence $A_n (= A)$.

We consider this definition as the most suitable, although there exist other methods for establishing deducibility relation $\Gamma \vdash A$, in logic, for which the concept of deduction differs structurally from that mentioned above. There are methods of sequent calculus, of analytical and semantic tables, resolution method, index method, etc. The result of each technique application allows an effective transforming into the standard form.

There are four types of problems that naturally follow from this definition.

1.1. Verifying the proof: $\Gamma \langle A_1, \ldots, A_n \rangle A$

Two sub-problems can be identified within this problem. The first sub-problem is substantiation of deduction rules, i.e. the preservation of useful properties (truthfulness) of formulas from antecedents to conclusions, which is usually solved by semantic means. The second sub-problem is checking the validity of deduction rules applications during their construction.

Based on the definition of proof, the problem of applying the validity check to deduction rules seems trivial, but only in theory. In reality the structure of mathematical proof is far from being ideally logical, which significantly
complicates checking the validity of already proven theorems and can cause an accumulation of undetected errors. For example, three years were spent checking the validity of the famous Perelman’s proof. Only a few people were able to fully understand the proof and confirm its validity. The rest of the mathematical community trusted them.

The well-known mathematician Vladimir Voevodsky has put forward a special research program of *Univalent Foundations of Mathematics* [10, 11]. One of the goals of the program was to develop a logic-mathematical language for making complex constructive proofs that could be easily verified by both humans and computers. He built a *homotopy type theory* to demonstrate the language of proof.

### 1.2. The search for interesting consequences: $\Gamma\langle\ldots\rangle$?

The second type of problems is the search for interesting consequences from the given proposition set $\Gamma$. A number of axioms from the fields of mathematics and natural sciences is known, yet it is not that obvious which new useful corollaries they may have. The development of methods that would allow us to get such corollaries is the dream of researchers in the field of artificial intelligence. However, one of the issues is that the set of consequences may be infinite, and therefore special conditions need to be stipulated to separate useful from non-useful consequences. This is a challenging task, but it may be of great theoretical and practical value.

V.A. Smirnov demonstrated the idea as applied to organic chemistry [17]. Like every branch of science, organic chemistry is based on some theoretical premises and related empirical data. A large but finite number of structural formulas are deduced based on the quantitative analysis of substances. Then these formulas are checked against the fundamentals of theory and empirical data. Many formulas are rejected, and those satisfying the requirements of theory and data are accepted.

### 1.3. Search for the proof: $\Gamma\langle\ldots?\ldots\rangle A$

This kind of problem is the most popular in logic. These are problems that underlie the axiomatic method of building scientific theories. There exists a set of axioms $\Gamma$, and we want to find if there exist proofs for specific formulas.

The fact that logic is mainly concerned with the deductability problem has some historical explanation. The idea of the axiomatic method goes back to Aristotle, who believed that there existed a small number of elements from which all true statements about the world at large could be deduced. The axiomatic method and proof search problem received a new development impulse at the turn of the 20th century. The crisis in mathematics required preserving most of the already gained mathematical results and
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their justification. Axiomatic systems were devised to concisely preserve the essence of existing mathematical theories, with the logical deduction giving the means for extracting indirect knowledge from these axioms. Since that time, the deduction search problem has become a major focus in logic, and the construction of a scientific theory based on the hypothesis-deduction method has become widely used in science.

Works in this area made it possible to obtain important meta-logical results on the properties of logic systems. However, despite high hopes, no single new theorem was deducted using proof search methods. Furthermore, as follows from Gödel’s theorems, the axiomatic approach to constructing scientific theories does not conform to the stipulated goals practically or theoretically. This is closely related to both the principal incompleteness of sufficiently rich theories and the impossibility of establishing their consistency. Aristotle’s idea on the possibility of explaining the variety of the surrounding world phenomena out of a small number of elements turned out to be principally impractical.

1.4. Search for hypotheses: \( A \)

The fourth type of problem is the search of plausible hypotheses, from which sentence \( A \) follows. Solving such problems is rarely studied in contemporary logic, although it has a more direct impact on research than the problem of proof search.

Syllogistics is believed to be one of the few exceptions. As we can see, enthymemes have a missing premise. In order to prove an enthymeme, it is necessary to find the missing premise and restore the full syllogism. As a result, with more knowledge gained, we learn which additional assumptions could or had to be made when analyzing the enthymeme in question.

On a practical level this does not concern enthymemes only. Let us consider the story of proving Fermat’s last theorem. Its proof had nothing to do with axiomatic deduction, as we might have imagined. First, in 1986 Ken Ribet demonstrated that Fermat’s last theorem could follow from Yutaka Taniyama’s hypothesis regarding the properties of elliptical curves over rational numbers. It was not until 1995 that Andrew Wiles proved this hypothesis. It is not exactly clear who should get credit for solving Fermat’s last theorem, although Andrew Wiles is officially considered to have been the one.

Another example are the famous Turing machines. The model was not a result of an ingenious revelation and it was not the result of a dream. Alan Turing thoroughly analyzed the operation of an abstract computer and formulated plausible hypotheses about its capabilities and available resources, then linked the two, using simple functional relations. The result was the axiomatic theory of Turing machines — abstract mechanical devices suitable for modeling any efficient calculations. This was not deduced from non-existing
axioms, but from analysis and further elaboration of what was considered to be an abstract computer.

The hypothesis search is associated with action planning problems. Suppose we need to build a *New Silk Road* from China to Europe. There are no axioms from which a statement of its existence could be deduced. There shall be a detailed analysis of what we mean by New Silk Road, an itemized problem decomposed into sub-problems, further decomposition of these sub-problems into sub-sub-problems specified until we get down to some basic elements of the plan that can be assigned to specific persons.

If logic in the process of finding proof is a *canon* and the obtained solution fulfills this canon, in the process of finding a hypothesis logic is an *organon* of new knowledge. We shall try to grasp both the code to present the final results, which can be based on a deductive approach, as well as logical methods for obtaining results.

An attractive characteristic of this problem type can be set forth as follows: since the set of hypothesis $\Gamma$ is not initially fixed, all these hypotheses do not fall under Gödel’s first incompleteness theorem. For each new specific statement $A$ we shall find the hypotheses $\Gamma$ based on which such statement could arise.

### 2. An Attempt to Reconstruction

Since finding the hypotheses $\langle\ldots\rangle A$ mainly rests upon the target sentence $A$, it would be quite natural to employ the analytical table method. However, the problem is that these methods can only be used when the analyzed sentence is the logic theorem, i.e. the set of hypotheses is empty; however, we are interested in finding such a set when it is not empty but unknown. That is why the analytical table method must be modified. The formula reduction rules used for table construction [8, 9] can be preserved, as they help transform a major problem to more simple sub-problems, depending on the problem logical structure.

Recall that analytical tables have the form of a tree of formulas. The table is closed, if and only if each tree branch contains a certain $C$ formula and its negation $\neg C$. According to the theorem of completeness, the classical logic formula $A$ is proved, if and only if there is a closed analytical table for its negation $\neg A$.

If all possible logical reduction rules are exhausted, and the table is not closed, in order to proceed we shall accept an additional formula $B$ which may relate to the current problem and appear in the target set $\Gamma$ in the future. The selection of $B$ shall not be made on an arbitrary basis, but based on a motivation. It seems to us that in the order of decreasing priority, the motivation for choosing formula $B$ may be related to this formula’s association with one of the following six types.
2.1 Formula $B$ can serve as a definition of a predicate or functional symbol which appears in the statement reduction obtained at the current step. The maximum priority to be assigned to explicit definitions can be explained by the fact that real problems are normally developed on the whole, and in order to solve them we shall specify the meaning of the definitions used. If we need to solve some problem, we can diode it into several sub-problems by adopting the corresponding definition.

However, no clarification made through definition can be final. Further we may need to continue to clarify the terms in the definiens. That is in general the reduction through definitions is a multi-stage process.

For example, if we are to prove Goldbach’s conjecture, we need to answer the question: What is it? In natural language it can be presented as follows: “Every even number more than two can be presented as a sum of two prime numbers”. Rigorous definition:

$$GB \equiv \forall x(x\in 2 & \text{Even}(x) \supset \exists y \exists z(\text{Prime}(y) \& \text{Prime}(z) \& x = y + z))$$

New descriptive symbols that appear in the definiens must also be defined:

$$\forall x(\text{Even}(x) \equiv \exists y(y > 0 \& x = y + y))$$
$$\forall x(\text{Prime}(x) \equiv x > 1 \& \forall y \forall z(x = y \times z \supset y = 1 \lor z = 1))$$

etc.

2.2 Formula $B$ can serve as an implicit axiomatic definition of a predicate or functional symbol which appears in the sentence reduction obtained at the current step. An axiomatic definition is a simple way to set recursive and inductive functions and predicates. As in the case for explicit definitions, this process can be split into several steps. For example, this can be due to the introduction of terms defined by non-fundamental and fundamental inductive definitions.

Using Goldbach’s conjecture as an example, we shall add implicit axiomatic definitions for $0$, $+$, $\times$ symbols.

2.3 $B$ may be a previously proved theorem. Currently there are a lot of proved arithmetic or geometric theorems, and we are not able to check all of them to find a premise. Only through analysis of the target sentence can we chose the proved theorems that will help us solve the problem.

Using Golbach’s conjecture, we identified all the symbols in its definition, however, we did not succeed in solving it. Here we can make use of previously proved theorems, for example, prime number distribution theorems.

2.4 $B$ can be an empirically true statement pertaining to the following problem: “The Volga runs into the Caspian Sea” or “Gold is yellow”. When we
solve problems related to planning actions, this could be actual information stored in databases.

**2.5** \( B \) can be a *description of some action’s results* pertaining to the given problem. For example, if we need to solve a geometrical problem, this could be a description of the result of auxiliary geometric construction. For the natural sciences, this may require performing an experiment and recording its result in \( B \). The description of an action’s results may be required when we need to solve an action planning problem.

**2.6** \( B \) may be a *plausible hypothesis* that can help solve the problem. This paragraph has the lowest priority, which is not the same as its significance. The assumption of plausible hypotheses is directly linked to the extension of our knowledge, and should therefore be used with caution. For example, a trace was discovered in a Wilson chamber whose trajectory may not be explained by any known elementary particle. This gives grounds for the deciding on the existence of a new particle. Specific hypothesis statements may be validated by inductive considerations, inference by analogy, etc.

In mathematics, a plausible hypothesis may be presented as a generalization of the initial statement, which also needs validation.

### 3. Method Formalization

Formalization will be performed as Fitting style analytical tables [8, 9], which are best suited for the formula reduction process. The main goal is to determine a general form of analytical reasoning and the necessary conditions to be met by its structure.

**Language initial symbols**

1. \( x, y, z, \ldots \) — a countable set of individual variables;
2. \( a, b, c, \ldots \) — a countable set of individual constants;
3. \( f^n, g^n, h^n, \ldots \) — a countable set of \( n \)-place functional symbols for every \( n > 0 \);
4. \( P^n, Q^n, R^n, \ldots \) — a countable set of \( n \)-place predicate symbols for every \( n \geq 0 \);
5. \( = \) — equality predicate;
6. \( \neg, \&, \lor, \supset \) — logical connectives;
7. \( \forall, \exists \) — quantifiers.
**Terms**

1. Any individual variable is a term;
2. Any individual constant is a term;
3. If \( t_1, \ldots, t_n \) are terms, and \( f^n \) is a \( n \)-ary functional symbol, then \( f^n(t_1, \ldots, t_n) \) is a term;
4. Nothing else is a term.

**Formulas**

1. If \( t_1, \ldots, t_n \) are terms, and \( P^n \) is a \( n \)-ary predicative symbol, then \( P^n(t_1, \ldots, t_n) \) is a formula.
2. If \( t_1 \) are \( t_2 \) are terms, \( t_1 = t_2 \) is a formula.
3. If \( A \) and \( B \) are formulas, then \( \neg A, (A \& B), (A \lor B), (A \supset B) \) are formulas.
4. If \( A \) is a formula, and \( x \) is an individual variable, then \( \forall x A, \exists x A \) are formulas.
5. Nothing else is a formula.

**Convention on designations**

1. We shall omit references to the arity of functional and predicate symbols as it can be derived from the context.
2. \( A \equiv B \) will be an abbreviation for the formula \((A \supset B) \& (B \supset A))\).
3. If \( S \) is an empty set of formulas and \( A \) is a formula, then we will use notation \( \{S, A\} \) or \( S, A \) as the abbreviation for \( S \cup A \).
4. \( A[P^n] \) denotes the formula \( A \) which has occurrences of the \( n \)-ary predicate symbol \( P^n \).
5. \( A[f^n] \) denotes the formula \( A \), which has occurrences of the \( n \)-ary functional symbol \( f^n \).
6. \( A[t] \) denotes the formula \( A \), which has occurrences of the term \( t \).
7. \( A[u/t] \) denotes the results of substitution of the term \( u \) in the formula \( A \) instead of selected occurrence of the term \( t \).
8. \( A(t/x) \) denotes the results of substitution of the term \( t \) in the formula \( A \) instead of all free occurrences of the variable \( x \).

9. \( FV(A) \) is a set of free individual variables of the formula \( A \).

10. \( FV(t) \) is a set of individual variables of the term \( t \).

11. A formula or term which has no free occurrence of variables shall be called a closed formula or term.

**Local reduction rules**

\[
\begin{align*}
(\neg \neg) & \quad S, \neg \neg A \quad \frac{}{S, A} \\
(\&) & \quad S, (A \& B) \quad S, A, B \\
(\neg \&) & \quad S, \neg (A \& B) \quad \frac{}{S, \neg A \mid S, \neg B} \\
(\lor) & \quad S, (A \lor B) \quad S, A \mid S, B \\
(\neg \lor) & \quad S, \neg (A \lor B) \quad \frac{}{S, \neg A, \neg B} \\
(\exists) & \quad S, \exists x A \quad S, \forall x A(t/x) \\
(\neg \exists) & \quad S, \neg \exists x A \quad S, \neg \forall x A \quad a \text{ is a new constant} \\
(\forall) & \quad S, \forall x A \quad S, A(t/x) \\
(\neg \forall) & \quad S, \neg \forall x A \quad S, \neg \exists x A \quad a \text{ is a new constant} \\
(\equiv) & \quad S, A[t/x], t = u \quad \frac{}{S, A[t], A[u/t], t = u}
\end{align*}
\]

**Restrictions on applying local reduction rules**

1. The term \( t \) in rules \((\forall)\) and \((\neg \exists)\) is an arbitrarily closed language term.

2. The terms \( t \) and \( u \) in rules \((=)\) and \((= /)\) are closed.
Global reduction rules

\[
(P) \quad \frac{A[P^n]}{B[P^n]} \quad (f) \quad \frac{A[f^n]}{B[f^n]}
\]

\[
(DP) \quad \frac{A[P^n]}{\forall x_1 \ldots \forall x_n P^n(x_1,\ldots,x_n) \equiv B} \quad FV(B) \subseteq \{x_1,\ldots,x_n\}
\]

\[
(Df) \quad \frac{A[f^n]}{\forall x_1 \ldots \forall x_n f^n(x_1,\ldots,x_n) = u} \quad FV(u) \subseteq \{x_1,\ldots,x_n\}
\]

Restrictions on applying global reduction rules

1. In the following rules \((P), (f), (DP)\) and \((Df)\) the formula under the line shall not contain individual constants that were previously introduced by the local reduction rules \((\exists)\) and \((\neg\forall)\).

2. As for the rule \((DP)\), here formula \(B\) does not contain the predicate symbol \(P^n\); in the rule \((Df)\) the term \(u\) does not contain the functional symbol \(f^n\).

It is clear that the rules \((DP)\) and \((Df)\) are two specials cases for the rules \((P)\) and \((f)\). However, we shall provide individual statements for them, as they carry a special meaning for the definition of predicative and functional symbols. The rules \((P)\) and \((f)\) contain a requirement that the same descriptive symbol occurs in formulas above and under the line, which means that a new added formula relates to the problem under consideration.

Concept of proof

Suppose \(U\) is a set of closed formulas.

The result of applying the rule \(R \in \{(\neg\neg), (\&), (\neg\lor), (\neg\supset), (\forall), (\neg\forall), (\exists), (\neg\exists), (=), (=\neq)\}\) to set of formulas \(U\) is its replacement by the set of formulas \(U_1\) if \(U\) coincides with the set of formulas above the line, while \(U_1\) coincides with the set of formulas under the line of this rule.

The result of applying the rule \(R \in \{(\neg\&), (\lor), (\ominus)\}\) to the set \(U\) is its replacement by the sets \(U_1\) and \(U_2\), if \(U\) coincides with the set of formulas above the line of the rule, while \(U_1\) and \(U_2\) coincide with two sets of formulas under the line.

We shall call the family of sets of formulas \(\{U_1, U_2, \ldots, U_n\}\) as configuration.
We shall call the result of applying the local reduction rule $R$ to the configuration $\{U_1, \ldots, U_n\}$ the replacement of this configuration with a new configuration, which has only one difference from the original configuration: it contains the result of applying the rule $R$ instead of one of the $U_i$ sets.

If at least one of the formula set in configuration $\{U_1, \ldots, U_n\}$ contains a formula above the line of the global reduction rule $R \in \{(P), (f), (DP), (Df)\}$, then the result of applying this rule $R$ to configuration $\{U_1, \ldots, U_n\}$ is new configuration $\{U'_1, \ldots, U'_n\}$ where each set $U'_i$ is obtained from $U_i$ by adding a formula under the rule $R$ line. That is global reduction rules change all configuration formula sets.

We shall call the table the finite sequence of configurations $\langle C_1, \ldots, C_n \rangle$, where every configuration, except the first one, is obtained from the preceding configuration as a result of local or definitional reduction rule.

The set of formulas $U$ is closed if contains a formula $A$ and its negation $\neg A$.

The configuration $\{U_1, \ldots, U_n\}$ is closed if every $U_i$ set is closed.

The table $\langle C_1, \ldots, C_n \rangle$ is closed if one of its configurations $C_i$ is closed.

We shall call the proof for the formula $A$ the closed table $\langle C_1, \ldots, C_n \rangle$ with initial configuration $C_1 = \{\neg A\}$. Clearly, if the formula $A$ is given as the implication $(B_1 \supset \ldots, (B_n \supset D))$, then, in order to prove it, we shall consider $C_1 = \{B_1, \ldots, B_n, \neg D\}$ as the initial configuration.

We shall assume that the formula $A$ is proved with regard to the Hyp hypothesis set if there is a closed table $\langle C_1, \ldots, C_n \rangle$ with the initial configuration $C_1 = \{\neg A\}$, and Hyp consists of all the formulas which were added by applying the rules $(P), (f), (DP)$ and $(Df)$ when developing this table.

We shall say that for the set of formulas $S$ there exists a closed table with regard to the Hyp hypothesis set, if there is a closed table $\langle C_1, \ldots, C_n \rangle$ with the initial configuration $C_1 = \{S\}$, and Hyp consists of all the formulas that were added by applying the rules $(P), (f), (DP)$ and $(Df)$ when developing this table.

4. Properties of Calculus Results

Theorem 1 states that global reduction rules are used to find hypothesis sets that are sufficient to construct a target sentence logical deduction, i.e. solution for the 4th problem type.

**Theorem 1.** (On the elimination of global reduction rules). If there is a closed table for the set of formulas $S$ with regard to the set of hypotheses Hyp =
\{B_1, \ldots, B_m\}, where \(m \geq 0\), then there is a closed table for the set of formulas \(S \cup \text{Hyp}\) without applying any global reduction rules.

**Proof.**

We shall prove by induction on the number of applications of the global reduction rules \(m \geq 0\).

*The induction basis* for \(m = 0\) is trivial.

*The induction step* for \(m > 0\), and we shall assume that proving the theorem also covers \(m - 1\).

Let \(\langle C_1, \ldots, C_i, C_{i+1}, \ldots, C_n \rangle\) be a closed table for the set of formulas \(S\) with \(m\) global reduction rules; we shall assume that in this set the configuration \(C_{i+1}\) was obtained from the configuration \(C_i\) as a result of the first application of one of these rules. This shall mean that if \(C_i = \{U_{i,1}, \ldots, U_{i,k}\}\), then the configuration \(C_{i+1} = \{U_{i,1} \cup \{B\}, \ldots, U_{i,k} \cup \{B\}\}\) for a formula \(B\) added by the use of this global rule.

Let us convert the table as follows. We shall replace every configuration \(C_j = \{U_{j,1}, \ldots, U_{j,r}\}\), where \(j \leq i\), with \(C'_j = \{U_{j,1} \cup \{B\}, \ldots, U_{j,r} \cup \{B\}\}\). The result is as follows:

1. The initial configuration \(C_1 = \{S\}\) is developed by \(C'_1 = \{S \cup \{B\}\}\).

2. Every configuration \(C'_j\), where \(1 < j \leq i\), is obtained from the configuration \(C'_{j-1}\) by the same local reduction rule which was applied when transferring from \(C_{j-1}\) to \(C_j\). This is true because adding one more formula to local reduction rule premises does not interfere with the rules. Problems may arise when using the rules \((\exists)\) and \((\neg \forall)\), but these problems can be eliminated as there is a restriction condition that formula \(B\) does not contain any individual constants that were previously added by these rules.

3. Configuration \(C'_i\) coincides with \(C'_{i+1}\).

In accordance with item 3, the configuration \(C'_{i+1}\) may be excluded and the proof with \(m - 1\) applications of global rules can be presented as follows: \(\langle C'_1, \ldots, C'_i, C_{i+2}, \ldots, C_n \rangle\).

\(\square\)

Theorem 2 allows to eliminate all explicit definitions introduced during construction of a proof.

**Theorem 2.** *(On the elimination of explicit definitions).* If the formula \((D_1 \& \ldots \& D_k \supset A)\) can be proved, where \(D_i\) is defined as
\[ \forall x_1 \ldots \forall x_n (P_i(x_1, \ldots, x_n) \equiv B_i) \text{ or } \forall x_1 \ldots \forall x_n (f_i(x_1, \ldots, x_n) = u_i), \] then we can also prove the formula \( A^* \) obtained from the formula \( A \) by expanding all assumed definitions.

**Proof.**

Let us consider the formula \( (D_1 \& \ldots \& D_k \supset A) \) and based on the theorem \( *352n \) [7, p. 183–184], depending on the type of \( D_i \), we shall make substitutions in the formula for all \( i \) from 1 to \( k \).

\[ S^P_{Bi}(x_1, \ldots, x_r)(D_1' \& \ldots \& D_k' \supset A') \]

or

\[ S^f_{ui}(x_1, \ldots, x_r)(D_1' \& \ldots \& D_k' \supset A') \]

After all the substitutions, we shall obtain the formula \( (D_1^* \& \ldots \& D_k^* \supset A^*) \), where every \( D^*_i \) formula is presented as \( \forall x_1 \ldots \forall x_r (B_i^* \equiv B_i^*) \) or \( \forall x_1 \ldots \forall x_r (u_i^* = u_i^*) \), and thereby \( D_1^* \& \ldots \& D_k^* \) is the predicate logic theorem.

From \( \vdash (D_1^* \& \ldots \& D_k^* \supset A^*) \) and \( \vdash D_1^* \& \ldots \& D_k^* \) by *modus ponens* we shall obtain \( \vdash A^* \). 

\[ \square \]

5. **Conclusion**

The paper [15] proposes a formal analytical approach to problem solutions based on the search for a model for the target sentence. As the problem of satisfiability of the formulas of the first-order predicate logics is not partially solvable, it was decided to confine it to finite models. Such a restriction is acceptable for many practical problems. In addition to the restriction by the finite domains, we selected the language which does not contain an equality predicate and functional symbols. This was done in order to facilitate the use of analytical reduction rules. The target sentence reduction was presented as a tree, and the problem is considered to be solved if at least one of the tree branches satisfied the closing conditions which allow to construct a model for target sentence.

In this work we consider the problems that assume a statement in the first-order predicate logic language without any additional restrictions on the models or language. The solution to the problem is obtained from the proof that states that the target sentence analytical table is closed if we assume some additional hypotheses. This approach is almost similar to the development of conventional analytical tables, but the difference is the application of global reduction rules. Like conventional analytical tables, the table is closed if all its branches are closed (one of the configurations is closed). Then, for the set of formulas introduced by the global rules, every model is a target sentence model. So, we obtain an adequate formalization required for solving Type 4 problems associated with the "conclusion" definition.
As for the modern logic which puts an emphasis on deduction, it should be noted that it acts as a **canon** — it states the way we should present our knowledge and the requirements for exact reasoning; however, it does not possess the **organon** instrumental functions — it is not a logic for finding new knowledge. Some attempts to equip the logic with these functions were implemented as part of the plausible reasoning method development process; however, no meaningful results were achieved. This devalues the logic value as a tool for researchers. Therefore, it is not surprising that the General Logic Course at the Moscow State University Faculty of Mechanics and Mathematics is only taught for one semester, while logic is not taught at all at other leading scientific faculties.

The analytical approach is not an alternative to the axiomatic-deductive method. They both solve their own specific problems. The axiomatic-deductive approach presents the knowledge when the knowledge is already obtained and we only have to present it in a suitable manner for further use. The analytical approach is based on obtaining new knowledge while solving new problems. Deduction methods play a secondary role; they are involved at the final stage when we need to demonstrate the obtained solution. Before this, the deduction methods and new assumptions guide researchers in their search for the hypotheses which further can be used for deducing the target sentence. As the set of hypotheses $\Gamma$ is originally not fixed, the restrictions of Gödel’s incompleteness theorem do not apply to the analytical approach. From any true but unprovable sentence of the available set of axiomatic statements we shall try to find additional axioms required to prove such a sentence. In view of the above, for the analytical approach logic is treated as an open system.

Modern logic cannot catch up with the knowledge that is being given at schools at a simple informative level [13] when two methods for problem solving are presented — **synthetic** and **analytical**. The synthetic method may be presented as follows:

*How do students usually solve a complicated problem? They pick up any given component from the problem statement and add some other given component to it. If these two components form a simple problem, they solve it; if it is not a simple problem, they develop another pair of components and, after solving the first simple problem, they obtain the first auxiliary given component. Using an auxiliary given component and another component of the other statements of the main problem, they solve the second simple problem and obtain the second auxiliary given component, etc., until they obtain such a simple problem the result of which is the desired value of the main problem. This is the synthetic method for problem solving [13, p. 180].*

*The synthetic method benefits from its compactness achieved through the statement of available solutions obtained by the synthetic or analytical process. Despite low*
search and didactic efficiency, the synthetic method is widely used by students and teachers, because it is very simple and does not require a great deal of thought [13, p. 184].

The quotation given above presents problem solving as a process going from the premises to the conclusion. The analytical method is described in a different way:

*For the analytical method, solutions are based on the requirement, the question, rather than the conditions of the problem specifications, as in the synthetic method. This is a common characteristic of all variations of the analytical method.* [...]

Solving a problem by the analytical method starts from posing a question related to the problem under consideration: “What do we need to know to answer the question of the given problem (to fulfill its requirement)?” [13, p. 185].

Then the author of the textbook considers actual examples that, according to the terms we adopted, correspond to the development of hypotheses in order to solve the problem.

In conclusion, it shall be noted that now it is high time to add one more topic to the standard logic courses — the theory of analytical problem solving. This work may be considered as a step towards the development of this theory.

References


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В настоящей работе рассматриваются четыре вида задач, которые естественным образом возникают в связи с определением логического вывода: 1) проверка доказательства: \( \Gamma(\text{A}_1, \ldots, \text{A}_n) \); 2) поиск интересных следствий: \( \Gamma(\ldots) \); 3) поиск доказательства: \( \Gamma(\ldots) \); 4) поиск гипотез: \( \text{A} \). В современной логике основное внимание уделяется задаче поиска доказательств. Ограничительные теоремы Гёделя имеют прямое к ней отношение. В то же время в реальной практике задача поиска гипотез, из которых следует целевое предложение, встречается гораздо чаще, чем задача поиска доказательства. Подробному ее исследованию и посвящена основная часть настоящей работы. Дается предложение \( \text{A} \), и требуется найти множество гипотез (посылок) \( \Gamma \), из которых оно логически выводимо. Выбор подходящих гипотез \( \Gamma \) происходит на основе логического анализа предложения \( \text{A} \). Мы можем выделить шесть различных оснований для выбора этих гипотез: 1) принятие явных определений для предикатных и функциональных символов; 2) принятие неявных аксиоматических определений для предикатных и функциональных символов; 3) принятие ранее доказанных теорем; 4) принятие эмпирически истинных предложений; 5) принятие предложений, описывающих результат некоторого действия; 6) принятие правдоподобных гипотез, которые могут иметь отношение к решаемой задаче. В работе построено исчисление, которое формализует задачу аналитического поиска гипотез для данного целевого предложения.

Ключевые слова: решение задач, логический вывод, поиск доказательства, поиск гипотез, логическая редукция, аналитические таблицы, теория определений

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