Elena B. Kuzina

On the concept of proof

Elena B. Kuzina
Lomonosov Moscow State University,
27/4 Lomonosovskiy prospect, Moscow, 119991, Russian Federation.
E-mail: elenakuzina@yandex.ru

Abstract: The term “proof” is used to refer to the whole spectrum of intellectual procedures aimed at establishing objective truth or at proving the truth of a certain sentence, the acceptability of the imperative, the fairness of evaluation, as well as convincing the other people of its adequacy. In mathematics, a proof plays a central role, but at the same time, there is not a general concept of mathematical proof. There are some very different perspectives on the nature of mathematical proof, its objectives, criteria and ideals, and over time these criteria and ideals change.

A proof in the other sciences is seen as a process of research, verification, and confirmation of certain provisions for the search and justification of objective or conventionally accepted truth. Here a proof consists essentially in searching for a supporting evidence, assessing it, and establishing that it proves a hypothesis best. A demonstrating reasoning, which is considered to be a proof in deductive sciences, need not be built up in many other areas.

In different areas of knowledge, criteria of viability and acceptability of evidence are different. In some it is a formal and deductive rigor, in the others it is an evidence of arguments and an intuitive clarity of reasoning, or it is a reliability and adequacy of a supporting evidence.

The main criterion for the admissibility of evidence is its credibility — the ability to cause a recipient to accept a proof of a statement so that he/she is willing to convince others. A proof is always immersed in a social and historical context, therefore, the concept of a proof that is common to all sciences and times not only does not exist but it cannot exist.

Keywords: truth, strictness, persuasiveness, supporting evidences, historical conditionality


The study outlines some of the most popular interpretations of the term of a “proof”.

(1) The most general concept of a proof is given in the Philosophical Encyclopedia, where a proof is defined as “a process of establishing the objective truth by practical and theoretical actions and means” [Eles, 1962, p. 42].

(2) In practical areas and in many fields of natural sciences, a proof is said to be a justification of the truth of a hypothesis by means of empirical data,
facts, and evidences. Moreover, the evidences themselves are denoted by the
term of a “proof”, too. This is the meaning of the term of a “proof” that it has
in law, history, biology, and many other sciences.

(3) In the traditional logical doctrine about a proof, it is said to be a process
or a method of a complete justification of the truth of some proposition or
some system of propositions by means of reasoning with the usage of other
propositions which truth has already been established.

(4) The narrowest reading of a proof is in formal theories, where a proof is
said to be a sequence of statements such that any statement is either a basic
postulate of the given theory, or is derived from them by the rules of reasoning
which are accepted in it.

(5) At last, a proof is said to be a verbal procedure which both is aimed at
convincing in and convinces an addressee in the truth of a certain statement
to the extent such that she/he accepts this statement and is ready to convince
the others with the help of the same procedure.

These five concepts which are connected to the term of a “proof” represent
three principally different approaches to a proof: first, as a search and a jus-
tification of the truth; second, as an intellectual game which is accepted by a
professional or another community; third, as a certain kind of a rational and
psychological impact on an addressee.

In any above-mentioned concepts of a proof, it is connected to the concept
of the truth, explicitly or implicitly. The different readings of a proof reflect
different conceptions of the truth: the classical one, the coherent one, and the
conventional one. The classical reading of the truth as an intentional consent
between the thought and the reality that exists independently of our conscious-
ness reflects in the most general, philosophical definition of a proof only.

Under the coherent reading of the truth as a coherence and an inconsistency,
a proposition is said to be true if it is an element of a logically inconsistent
and coherent system. This conception of the truth is expressible itself in all
the definitions of a proof, where a proof is understood both as a justification
of the truth of a proposition by means of the other propositions and as an
establishment of logical connections between it and the other propositions. In
this sense, a proof is, in essence, an integration of a justifying statement into
the accepted system of knowledge.

The conventional conception of the truth considers knowledge being true
if there is an obtained consent about it; therefore, a proof is supposed to be
understood as a process of motivating to the consent and accepting the proposed
statement by an addressee, i. e. as a process of persuading. Hence, the latter of
the above-mentioned readings of the term of a “proof” reflects the conventional understanding of the truth, to some extent.

In the vast majority of people’s view, mathematical proofs are the etalon. The concept of a proof does not just play the central role in mathematics; it seems to express the essence of mathematics.

However, it is not quite clear what is a mathematical proof, and there are different opinions about it. At least, one might count four of them. They have different views on goals of a mathematical proof; they reflect different readings of the mathematical truth, as well. A mathematical proof might be read as follows:

1. an establishment of the immutable and undoubted truth of a mathematical statement;

2. an explication of the meaning of a mathematical statement and a reduction of it to an obviousness;

3. a proof is a language-game playing by the given rules;

4. a way to convincing the others, the mathematical community, above all, in the truth of a mathematical statement.

I. The followers of the position that a proof justifies the truth in the classical sense assume the existence of a certain mathematical reality that is cognizable. The correspondence between a proposition and this reality is justified by a proof. According to this point of view, a mathematical proof is grounded into certain apodictic obviousnesses, which are unchanged and are the same for all moments of time, all cultures, and all languages. And that is why if it is achieved, then no one ever could doubt in it. “The vast majority of proofs accepted by mathematicians, V.Ya. Perminov states, has a total reliability which could not be shaken by any changes in the given mathematical theory and in mathematics generally”. The reliability of a mathematical proof is absolute, but its strictness is always relative. Its criteria are historically changeable; every epoque has its own criteria of a strictness and its requirements to logic of a proof [Perminov, 2013, p. 76].

II. The second point of view which states that the goal of a proof is both to explicate the sense of a mathematical statement and to achieve a deeper and fuller understanding of the object, assumes certain obviousnesses as a foundation of a proof, as well. However, achieving the clearness of mathematical concepts and propositions is sometimes connected to the fact that some statements which are known from experience, i. e. assertoric ones, are
accepted as obvious. In order to make intuitive representations strict, one tries to reduce them to formal expressions that does not just make a proof stricter, but more complex and less intuitively clear, as well [Gutner, 2013, pp. 143–144].

III. The reading of a mathematical proof as a language-game dates back to Hilbert’s program with its idea to find, for any separately given field of mathematics, a collection of axioms and rules of inference which would be sufficiently complete for any possible mathematical reasoning that is valid in this field. An intention to both the strictness of a mathematical proof and the liberation of it from experienced, assertoric obviousnesses that realizes via a formalization of all foundations of a proof turns it into a language-game by the given rules and brings it closer to formal proofs in logical calculi. Every formalization of the basic intuitions is conventional; it is supposed to be accepted as adequate by the scientific community. The scientific community considers a mathematical statement to be proven if it is grounded into the accepted formalized postulates and is built upon the accepted rules.

IV. The understanding of a mathematical proof as a way to convince oneself and the other members of the professional community in the truth of an assuming statement dominates among philosophers of mathematics. According to this point of view, the essence of a mathematical proof is in its persuasiveness such that a person who accepted it is ready and able to convince the others. A proof is a form of appealing to the scientific community and, therefore, directly depends on the norms of reasoning, assessments, and opinions which are accepted in this community [Bazhanov, 2013, p. 50]. A proof in mathematics does not differ, in essence, from proofs in the other sciences: its task is to convince; the threshold of persuasiveness of a proof in mathematics is simply higher than the one in the other sciences or practical spheres. As V.A. Uspensky puts it, non-mathematical proofs pretend to be convincing in that a proving statement takes place with a very high probability, and an assumption that it is not the case is improbable. Mathematical proofs yet pretend to be convincing in that a proving statement takes place necessarily, and an assumption that it is not the case is impossible [Uspenskij, 2009, p. 6].

A proof in all the other fields of science and practical activity is, mostly, in making arguments or “evidences” in the favour of a proving statement and is not in building up a reasoning. Being steadily increased, positive results of the test decreases the probability of the fallacy of a hypothesis. If a total “persuasiveness” of the collected evidences in the favour of a justifying statement increases to the extent that competent scientists just have no more reasons to be doubt-
ful in its validity, then one starts to consider it as the proven truth [Markov, 2018]. A demonstrating reasoning that is a proper proof in mathematics and logic does not play the decisive or even independent role in natural sciences as well as in history and law.

Both the intuitive clearness and the visibility of a proof play the decisive role that, in my mind, is directly connected to persuasiveness. The strength of persuasiveness of every separate “evidence in favour” is definable by the extent that it is reliable and concrete. The inner belief in the truth of a proven statement which is discussed about in law is in comprehending the impossibility of the contrary or some other view on the disputed question.

Habitually, we operate with the ambiguous and quite vague term of a “proof” as if there is a strictly defined concept of a proof, where the ideal of a rational justification is expressed clearly. However, there is no such a concept which holds for every epoque and every field of knowledge. One always considers a proof to be the thing that convinces the competent addressee.


References


Uspenskij, 2009 – Uspenskij, V. A. Prostejshie primery matematicheskikh dokazatel’stv
[The simplest examples of mathematical proofs]. Moscow: MTsNMO, 2009. 56 pp. (In Russian)
О понятии доказательства*

Елена Борисовна Кузина
МГУ им. М.В. Ломоносова.
Российская Федерация, 119991, г. Москва, Ломоносовский пр-т, д.27, корп.4.
E-mail: elenakuzina@yandex.ru

Аннотация: Термин «доказательство» используется для обозначения целого спектра интеллектуальных процедур, направленных на установление объективной истины или обоснование истинности некоторого предложения, приемлемости императива, справедливости оценки, а также на убеждение других людей в его адекватности. В математике доказательство играет центральную роль, но вместе с тем общего понятия математического доказательства нет. Существует несколько весьма различных точек зрения на сущность математического доказательства, его цели, критерии и идеалы, и со временем эти критерии и идеалы меняются.

Доказательство в других науках рассматривается как процесс исследования, проверки и подтверждения некоторых положений с целью поиска и обоснования истины — объективной или конвенционально принятой. Здесь доказательство заключается главным образом в поисках подтверждающих свидетельств, их оценке и установлении того, что лучше всего они объясняются доказываемой гипотезой. Построение демонстрирующего рассуждения, которое и считается доказательством в дедуктивных науках, во многих других областях совсем не обязательно.

В разных областях познания критерии состоятельности и приемлемости доказательств различны. В одних — это формально-дедуктивная строгость, в других — очевидность аргументов, интуитивная ясность рассуждения, в третьих — достоверность и достаточность подтверждающих свидетельств.

Основным общим критерием приемлемости доказательства представляется его убедительность — способность вызвать у адресата такое принятие доказанного утверждения, что он готов убеждать в нем других. Доказательство всегда погружено в социально-исторический контекст, поэтому общего для всех наук и всех времен понятия доказательства не только не существует, но и не может существовать.

Ключевые слова: истинна, строгость, убедительность, подтверждающие свидетельства, историческая обусловленность


Литература


