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Questions to Michael Dunn

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Abstract: We present nine questions related to the concept of negation and, in passing, we refer to connections with the essays in this special issue. The questions were submitted to one of the most eminent logicians who contributed to the theory of negation, Prof. (Jon) Michael Dunn, but, unfortunately, Prof. Dunn was no longer able to answer them. Michael Dunn passed away on 5 April 2021, and the present special issue of *Logical Investigations* is dedicated to his memory. The questions concern (i) negation-related topics that have particularly interested Michael Dunn or to which he has made important contributions, (ii) some controversial aspects of the logical analysis of the concept of negation, or (iii) simply properties of negation in which we are especially interested. Though sadly and regrettably unanswered by the distinguished scholar who intended to reply, the questions remain and might stimulate answers by other logicians and further research.

Keywords: Michael Dunn, Negation, Belnap-Dunn logic, American Plan, Australian Plan, Paraconsistency, Negation inconsistency, Rule of contraposition, Nelson’s constructive logics with strong negation, Negation as cancellation, *Ex contradictione nihil sequitur*, demi-negation, Proof-theoretic bilateralism

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1. Introduction

Instead of writing a detailed introduction to this special issue of *Logical Investigations* and thereby putting some timely research issues into perspective, the guest-editors decided to directly delve into the topic by posing a number of questions to one of the world-wide leading experts on the logical study of the concept of negation, Prof. J. Michael Dunn. The questions address focal and partly contentious issues related to negation. Unfortunately, Prof. Dunn was no longer able to answer the questions submitted to him. Michael Dunn passed away on 5 April 2021, and we decided to dedicate the present special issue of *Logical Investigations* to his memory and to present the unanswered questions. We hope they stimulate replies by other logicians interested in the notion of negation.

Prof. J. Michael Dunn (1941–2021)

2. Nine questions to Michael Dunn concerning negation

Michael Dunn has been known as one of the central and major contributors to Relevance Logic. Systems of relevance logic are paraconsistent logics since they reject the idea of *ex contradictione quodlibet* both as an inference, $\{A, \neg A\} \vdash B$, and as a theorem in formal languages with conjunction and implication, $(A \wedge \neg A) \rightarrow B$. Moreover, Dunn was one of the developers of a system that is frequently called *Belnap-Dunn logic*, or *first-degree entailment logic*, a fundamental and especially natural four-valued paraconsistent logic that has found many applications in various areas ranging from the philosophy of information to artificial intelligence, see, for example, [Omori and Wansing,

2017]. According to Graham Priest, there exists a slippery slope that leads from the endorsement of a paraconsistent logic, in order to enable non-trivial inconsistent theories, to a metaphysical position, *dialetheism*, according to which there exist true contradictions. Of course, advocates of a system of paraconsistent logic need not be dialetheists, yet it is interesting to see how scholars who investigate paraconsistent logics think about dialetheism. This prompted our first question to Michael Dunn, **Q1**. The topics of paraconsistency and relevance figure prominently in Arnon Avron’s contribution to this special issue *Implication, equivalence, and negation*.

Q1 Can you briefly comment on your general attitude towards *paraconsistent logic* and *dialetheism*, and how would you locate the *Belnap-Dunn logic*, or the *Sanjaya-Belnap-Smiley-Dunn Four-valued Logic*, as you suggested in [Dunn, 2019], in your picture?

Some well motivated expansions of Belnap-Dunn logic turned out to be non-trivial *contradictory* logics, such as the logic of logical bilattices investigated by Ofer Arieli, Arnon Avron, Melvin Fitting, and other logicians. Negation inconsistency is a very remarkable property that makes these systems orthogonal to classical logic. According to Karl Popper, [Popper, 1962, p. 322], “The acceptance of contradictions must lead ... to the end of criticism, and thus to the collapse of science.” If contradictions are provable in the logic on which a scientific theory is based, this is certainly a challenging feature. How are we to think about this thought-provoking property? Well, we wanted to know how Michael Dunn thought about it, **Q2**.

Q2 The *Belnap-Dunn logic* has been extremely fruitful in considering some expansions, such as the *logic of bilattices*, as well as the *connexive logic C*. We highlight these because of their shared feature of being *negation inconsistent*, namely for some formulas, both the formula and its negation are valid/derivable. Do you have any thoughts on this kind of *inconsistent logics*?

A semantics in terms of information states for a non-trivial contradictory logic has it that not only a state may both support the truth as well as the falsity of one and the same formula, but that there are formulas such that *every* state supports both their truth and their falsity. Thomas Studer in his paper *A conflict tolerant logic of explicit evidence* does not exactly take such a bold step. He presents a justification logic, CTJ, that accepts two different pieces of evidence such that one justifies a proposition whereas the other piece of evidence justifies the negation of that proposition. Yet, CTJ has no room for pieces of evidence that justify both a proposition and its negation. If one

would think of justifications in terms of proofs in a logical system, however, two formulas A and $\neg A$ may both have different proofs.

Paraconsistency is a property a logical system has or fails to have chiefly in virtue of a negation connective occurring in its language. The famous paraconsistent Belnap-Dunn logic enjoys several semantical characterizations. In particular, negation can be captured by means of a four-valued semantics but also by means of a two-valued semantics. Negation on the so-called “Australian plan” models the negation connective as a point shift operator in a two-valued semantics, famously especially in the Routley star semantics. The four-valued approach of the so-called “American plan” can be combined with a state semantics, but negation essentially flip-flops (support of) truth and falsity at a given state, i.e., point of evaluation. Questions **Q3** and **Q4** address the two plans.

Q3 Here is another question continuing with the *Belnap-Dunn logic*. Given that there are two semantics for negation in the *Belnap-Dunn logic*, the *American plan* and the *Australian plan*, and that you have contributed immensely to the developments of both plans, we are interested in your basic picture concerning negation. Is the following quote from one of your earlier papers something you are willing to defend?

Tim Smiley once good-naturedly accused me of being a kind of lawyer for various non-classical logics. He flattered me with his suggestion that I could make a case for anyone of them, and in particular provide it with a semantics, no matter what the merits of the case [...]. But I must say that my own favourite is the 4-valued semantics. I am persuaded that ‘ $\neg\phi$ is true iff ϕ is false’, and that ‘ $\neg\phi$ is false iff ϕ is true’. And now to paraphrase Pontius Pilate, we need to know more about ‘What are truth and falsity?’. It is of course the common view that they divide up the states into two exclusive kingdoms. But there are lots of reasons, motivated by applications, for thinking that this is too simple-minded. [Dunn, 1999, p. 49]

Q4 A question related to the previous question concerns the recent discussions of the Australian plan of negation defended by Francesco Berto and Greg Restall in [Berto, 2015; Berto and Restall 2019] and criticized in [De and Omori, 2018]. What might be the lesson we should learn from the Australian plan, if one’s preferred approach is the American plan?

Suppose we want to have a conditional that validates modus ponens and the deduction theorem. Then the two plans in many cases fall apart. Whereas the contraposition rule is valid on the Australian plan, on the American plan, in the presence of such an implication connective, the contraposition rule can be easily invalidated. The validity of contraposition is but one of several properties that have been discussed as characteristic features of negation. The double negation laws, the De Morgan laws, and the Law of Excluded Middle have also been discussed as criteria of negationhood. The latter principle is at center stage in the invited contribution by Jc Beall and Graham Priest, *A tale of excluding the middle*, in which the authors discuss an argument for the dialethic nature of the liar sentence. Our next question, **Q5**, turns to properties of negation.

Q5 Continuing from the previous question, some defend the *rule of contraposition*, namely if $A \vdash B$ then $\neg B \vdash \neg A$ as the essential rule for negation, but as you have taught all of us, this rule requires some very delicate treatment in the context of the *Belnap-Dunn logic*. What is your opinion on the *rule of contraposition*? Do you think any of the properties of negation will stand out as essential? Do you also have thoughts on the minimal requirement for negation, i.e. for some atomic formulas p and q , $p \not\vdash \neg p$ and $\neg q \not\vdash q$, suggested by Wolfgang Lenzen [Lenzen, 1996] and João Marcos [Marcos, 2005] and later adopted by Ofer Arieli, Arnon Avron, and Anna Zamansky in [Arieli et al., 2011]?

Prof. J. Michael Dunn and Prof. Nuel D. Belnap
Pittsburgh, April 2018
(Photo: courtesy of Prof. Anil Gupta)

In §1 of *Entailment. Vol. I*, Alan Anderson and Nuel Belnap explain that they “take the heart of logic to lie in the notion ‘if ... then –’”. It may therefore seem especially significant to consider the relationship between implication and negation. If the negation connective is meant to express falsity, then the question about the relationship between implication and negation boils down to asking about the falsity conditions of implications. That’s what question **Q6** is doing by contrasting the negation in David Nelson’s constructive logics and the understanding of negated implications in certain (hyper)connexive logics.

Q6 Given the developments related to the relevant logic **R**, for example, there are some clear interactions between the treatment of negation and the treatment of implication. Do you have any thoughts how we should think about the interactions? Would there be any guiding principles for you to think about the interactions? In particular, in the Kripke semantics for David Nelson’s constructive logics with strong negation, usually denoted by “ $\sim$ ”, we have the classical falsity condition for negated implications insofar as a state verifies $\sim(A \rightarrow B)$ iff it verifies A and falsifies B , so that $\sim(A \rightarrow B) \leftrightarrow (A \wedge \sim B)$ is valid. In certain connexive logics, however, the falsity condition of implications is such that $\sim(A \rightarrow B) \leftrightarrow (A \rightarrow \sim B)$ is valid, cf. [Omori and Wansing, 2019; Wansing, 2020]. Do you have an opinion on this matter?

The non-involutive weakening of strong negation in the three-valued para-complete version of Nelson’s logic is investigated in the contribution by Thiago Nascimento da Silva and Umberto Rivieccio, *Negation and implication in quasi-Nelson logic*, where quasi-Nelson algebras are presented as a generalization of both Heyting algebras and Nelson algebras.

Question **Q6** gives rise to a plethora of further considerations. One of the semantical assumptions that underlie classical logic is bivalence, the view that there are exactly two truth values, *true* and *false*, and that in a given situation every meaningful statement takes exactly one of these values. In many-valued logic, this is a subtle issue as a distinction can be drawn between algebraic and inferential values, where the latter are usually given by a bi-partition of the set of algebraic values into a non-empty set of designated values and its nonempty complement, cf. [Malinowski, 1993; Malinowski, 1994; Malinowski, 2009; Blasio et al., 2017]. It is the latter sets of values that are used in definitions of semantical consequence. In a bivalent setting, a statement is false exactly when it is not true. In a many-valued setting, however, being untrue, respectively, non-designated, and being false, respectively anti-designated, fall apart. Gilberto Gomes in his contribution *Negation of conditionals in natural language and thought*, discusses the external negation of natural language conditionals

expressed by means of “it is not the case that” and “it is not true that”, keeping in mind a distinction between implicative and concessive conditionals. Gomes concludes that in English conditionals *If A, then B* entail *It is not the case that if A, then not B*, but presents counterexamples to the converse. One may wonder whether native speakers of English are ready to draw a distinction between “it is not the case that *A*” and “it is definitely false that *A*”.

The non-contraposible negation as falsity in Nelson’s logics is but one type of negation, and several other ways of trying to capture semantic opposition by means of a unary sentential operator or formalizing negation phenomena in natural languages are known from the literature. Questions **Q7** and **Q8** address two such notions: negation as cancellation and demi-negation.

Q7 Quite a few different kinds of negation have been distinguished between in the literature. We would like to know your opinion on some of those listed in the *Stanford Encyclopedia of Philosophy* entry on negation [Horn and Wansing, 2020]. One such concept is the notion of negation as cancellation or erasure that has been discussed by Richard and Valerie Routley [Routley and Routley, 1985] and Graham Priest [Priest, 1999], and that has been very heavily criticized more recently in [Wansing and Skurt, 2018]. The Routleys [Routley and Routley, 1985, p. 205] characterize negation as cancellation as follows:

$\sim A$ deletes, neutralizes, erases, cancels A (and similarly, since the relation is symmetrical, A erases $\sim A$), so that $\sim A$ together with A leaves nothing, no content. The conjunction of A and $\sim A$ says nothing, so nothing more specific follows. In particular, $A \wedge \sim A$ does not entail A and does not entail $\neg A$.

This idea is closely related to the slogan *ex contradictione nihil sequitur* (nothing follows from a contradiction), see [Wagner, 1991]. Do you think that this is a reasonable and viable conception of negation?

Q8 There is also the notion of negation by iteration, i.e., the idea to obtain a negation by a double application of a connective called “demi-negation” in [Humberstone, 1995], or “square root of negation”, $\sqrt{\text{not}}$, in quantum computational logic, e.g. in [Paoli, 2019]. In [Omori and Wansing, 2018] it is speculated that double demi-negation as negation could be used to analyze the phenomenon of negative concord in certain natural languages (or natural language dialects) such as in “She don’t eat no biscuit”, and it is discussed whether certain demi-negations are indeed negations. Have you ever thought about demi-negations?

The proper context of *ex contradictione nihil sequitur* is non-monotonic reasoning. If one wants to keep the reflexivity of the derivability relation, then, obviously, blocking the step from $A \vdash A$ to $A \wedge \neg A \vdash A$ will make inferences non-monotonic. Yet another type of negation is negation as failure to derive. It has been clear that logic programming needs in addition to negation as failure as a non-monotonic inference rule another kind of negation, explicit negation. Both types of negation are studied in Reinhard Kahle’s paper *Default negation as explicit negation plus update*.

In the last question, **Q9**, the attention is directed to the treatment of negation in proof-theoretic semantics. The term “proof-theoretic semantics” was coined by Peter Schroeder-Heister, and not the least through his efforts, proof-theoretic semantics is now an established and very active research area. Like other areas within logic, it has seen a certain preoccupation with positive notions such as assertion and verification, but when negation comes into focus, their negative counterparts are in the spotlight. One logical system, in which there is a kind of duality between verification and refutation and between implication and a concept of co-implication is Heyting-Brouwer logic, also known as bi-intuitionistic logic, **BiInt**. Whereas in its Kripke semantics the co-implication of **BiInt** is a backwards-looking existential quantifier, the co-implication of the likewise bi-intuitionistic logic **2Int** is a forward-looking universal quantifier. Paolo Maffezioli and Luca Tranchini in their contribution *Equality and apartness in bi-intuitionistic logic* discuss the concepts of identity and apartness in the context of Heyting-Brouwer logic.

Q9 Let us finally come to the interplay between negation and consequence relations. In your paper “Partiality and its Dual” [Dunn, 2000] and elsewhere, you consider various semantically defined consequence relations that differ in whether valuations permit truth-value gaps, gluts, or both. The permission of gaps, gluts, or both is certainly a negation-related topic. One of the systems you deal with in the mentioned paper is a well-known expansion of *Belnap-Dunn logic*, viz. Almukdad and Nelson’s constructive four-valued logic, nowadays known as **N4**. On the proof-theoretic side, **N4** has been arrived at via different roads, one being what is often called “bilateralism,” the view that concepts like falsity, denial, and refutability should be considered in their own right and as being as important as their positive counterparts truth, assertion, and provability. As a result, this approach may lead one to making use of two separate derivability relations: provability and disprovability (or refutability). In that context, it has also been suggested, e.g. in [Wansing, 2013; Wansing, 2017; Drobyshevich, 2019], to consider in addition to implication as a connective that internalizes in the logical object language the preservation of (support

of) truth, a co-implication connective that internalizes preservation of (support of) falsity. The meaning of a logical operation is then to be specified proof-theoretically by its inferential roles in *both* proofs and disproofs. What would you say about such a conception of meaning?

Prof. J. Michael Dunn meeting for lunch
with two of the guest editors during
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